

Axial exchange currents and nucleon spin

D. Barquilla-Cano¹, A.J. Buchmann^{2,a}, and E. Hernández¹

¹ Grupo de Física Nuclear, Facultad de Ciencias, Universidad de Salamanca, Plaza de la Merced s/n, E-37008 Salamanca, Spain

² Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

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Abstract. We calculate the axial couplings $g_A^8(0)$ and $g_A^0(0)$ related to the spin of the nucleon in a constituent quark model. In addition to the standard one-body axial currents, the model includes two-body axial exchange currents. The latter are necessary to satisfy the Partial Conservation of Axial Current (PCAC) condition. For both axial couplings we find significant corrections to the standard quark model prediction. Exchange currents reduce the valence quark contribution to the nucleon spin and afford an interpretation of the missing nucleon spin as orbital angular momentum carried by nonvalence quark degrees of freedom.

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1 Introduction

The question which degrees of freedom carry which part of the nucleon's spin has stimulated a great deal of research on both the experimental and theoretical side [1]. In a quantum field theoretical description [2] the nucleon spin J is built from the quark spins Σ , quark orbital angular momentum L_q , gluon spin Δg , and from gluon orbital angular momentum L_g as

$$J = \frac{1}{2} \Sigma + L_q + \Delta g + L_g = \frac{1}{2}. \quad (1)$$

Presently, there is still considerable controversy over how the nucleon spin $J = 1/2$ is made up in detail. In the standard constituent quark model [3], which uses only one-body axial currents, one obtains a clear answer, namely $J = \Sigma/2 = 1/2$, *i.e.*, the nucleon spin is the sum of the three constituent quark spins and nothing else. On the other hand, in the Skyrme model [4] most of the nucleon spin is due to orbital angular momentum of the quarks. Experimental results [5,6] for the quark spin sum indicate that $\Sigma/2 = 0.16(5)$, *i.e.*, only 1/3 of the total angular momentum of the nucleon is due to quark spins. This marked disagreement between the standard constituent quark model and experiment has often been interpreted as a severe shortcoming of this model (“spin crisis”).

However, the simple additive constituent quark model violates the partial conservation of the axial current

(PCAC) condition, which is an important symmetry constraint that must be satisfied in a consistent theory. The purpose of this paper is to show that if the constituent quark model is supplemented by two-body axial exchange currents so that the PCAC condition is satisfied, the valence quark contribution to the nucleon spin is considerably reduced. Furthermore, the model suggests that the nucleon contains a substantial amount of orbital angular momentum carried by nonvalence quark degrees of freedom effectively described by exchange currents.

The paper is organized as follows. In sect. 2 we review the relation between the axial current matrix elements and the spin fraction carried by quarks. The axial current operators of the constituent quark model are briefly discussed in sect. 3. Our results for the nucleon spin structure are presented in sect. 4 and summarized in sect. 5.

2 Spin structure of the nucleon

Experimentally, information on the spin structure of the nucleon is obtained from deep inelastic scattering (DIS) of polarized leptons on longitudinally polarized protons. In such an experiment one is measuring the spin-dependent proton structure function $g_1^p(x, Q^2)$, which depends on the four-momentum transfer $Q^2 = -q_\mu q^\mu$ and the Bjorken scaling variable $x = Q^2/(2M\nu)$ [7]. At Q^2 and ν large compared to the nucleon mass M (scaling limit), the integral of the spin structure function can be calculated in

^a e-mail: alfons.buchmann@uni-tuebingen.de

the framework of QCD to first order in the strong coupling constant α_s [8]:

$$\begin{aligned} \Gamma_1^p(Q^2) &=: \int_0^1 g_1^p(x, Q^2) dx \\ &= \left(\frac{4}{18} \Delta u(Q^2) + \frac{1}{18} \Delta d(Q^2) + \frac{1}{18} \Delta s(Q^2) \right) \\ &\quad \times \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right), \end{aligned} \quad (2)$$

where the $\Delta q(Q^2)$ are the fractions of the nucleon spin carried by the quarks and antiquarks of flavor $q = (u, d, s)$ evaluated at the renormalization scale $\mu^2 = Q^2$. In the parton model, the spin fractions carried by the individual quark and antiquark flavors are defined as

$$\begin{aligned} \Delta q(Q^2) &= \int_0^1 dx (q \uparrow(Q^2, x) + \bar{q} \uparrow(Q^2, x) \\ &\quad - q \downarrow(Q^2, x) - \bar{q} \downarrow(Q^2, x)). \end{aligned} \quad (3)$$

The quark momentum distributions $q \uparrow(Q^2, x)$ and $q \downarrow(Q^2, x)$ denote the probability for finding in the nucleon a current or QCD quark [9] of flavor q with momentum fraction x of the total nucleon momentum and spin parallel or antiparallel to the proton spin. The spin quantization axis is chosen along the proton momentum. The quark momentum distributions, *e.g.*, $q \uparrow(Q^2, x)$, can be further decomposed:

$$q \uparrow(Q^2, x) = q_{val} \uparrow(Q^2, x) + q_{sea} \uparrow(Q^2, x), \quad (4)$$

where $q_{val} \uparrow(Q^2, x)$ and $q_{sea} \uparrow(Q^2, x)$ is the contribution of the QCD valence and sea quarks. Note that the definition of Δq in eq. (3) also contains the antiquark distributions denoted by $\bar{q} \uparrow(Q^2, x)$ and $\bar{q} \downarrow(Q^2, x)$. Hence, the spin fractions involve both valence quarks and sea quark-antiquark pairs.

The fraction of the nucleon spin fractions carried by the individual quark flavors q can be expressed as axial vector current matrix elements

$$\Delta q(\mu^2) = \langle p \uparrow | g_{Aq} (\bar{q} \gamma_3 \gamma_5 q) | \mu^2 | p \uparrow \rangle, \quad (5)$$

where $\gamma_3 = \gamma_z$ and γ_5 are Dirac matrices, μ^2 refers to the mass scale at which the axial current operator is renormalized, and g_{Aq} is the quark axial current coupling constant.

In the constituent quark model one does a tree level calculation with the usual assumption that the quark fields q in eq. (5) are described by free Dirac spinors. In addition, the static limit is taken so that the contribution from the lower components of the Dirac spinors can be neglected. Furthermore, it is assumed that $g_{Aq} = 1$, *i.e.*, the same as for structureless QCD quarks, in which case the axial current matrix element reduces to

$$\Delta q = \langle p \uparrow | \sigma_z^q | p \uparrow \rangle. \quad (6)$$

Thus, in this approximation, the axial quark current is the one-body [10] quark spin operator σ_z^q . Its matrix element between nucleon states measures the contribution of

a valence quark with flavor q to the nucleon spin, where q is restricted to the u and d flavors. The total angular momentum of the nucleon is then simply the sum of the u - and d -quark spin fractions. With the usual $SU(6)$ spin-flavor wave function of the proton and the one-body axial current operator in eq. (6), one obtains for the spin fractions $\Delta u = 4/3$ and $\Delta d = -1/3$ adding up to

$$\langle J_z \rangle = \langle S_z \rangle = \frac{1}{2} (\Delta u + \Delta d) = \frac{1}{2}. \quad (7)$$

More generally, allowing for three flavors in the axial current operator in eq. (5), and evaluating matrix elements for $SU(3)$ flavor octet baryons, one finds that the three axial form factors at zero momentum transfer, $g_A(0)$, $g_A^8(0)$, and $g_A^0(0)$ are the relevant quantities that carry the information on the nucleon's spin structure. The relations between the axial couplings g_A and the spin fractions Δq are

$$\begin{aligned} g_A(0) &= \Delta u(Q^2) - \Delta d(Q^2), \\ g_A^8(0) &= \Delta u(Q^2) + \Delta d(Q^2) - 2\Delta s(Q^2), \\ g_A^0(0)_{Q^2} &= \Delta u(Q^2) + \Delta d(Q^2) + \Delta s(Q^2) =: \Sigma. \end{aligned} \quad (8)$$

The flavor octet $g_A(0)$ (isovector) and $g_A^8(0)$ (hypercharge) couplings are independent of the renormalization point. On the other hand, the flavor singlet axial coupling $g_A^0(0)$ depends on the renormalization scale at which it is measured [11], hence the subscript Q^2 .

Experimental results for $g_A(0)$ and $g_A^8(0)$ have been obtained from the weak semileptonic decays of octet baryons [12–15]. With $\Gamma_1^p(Q^2)$ measured in DIS [5] and the axial couplings $g_A(0)$ and $g_A^8(0)$ determined from β decays, there are three experimental data and three unknown spin fractions Δq . The combined DIS and hyperon β -decay data give [5]

$$\begin{aligned} \Delta u &= 0.83 \pm 0.03, & \Delta d &= -0.43 \pm 0.03, \\ \Delta s &= -0.09 \pm 0.04. \end{aligned} \quad (9)$$

The sum of these experimental spin fractions $\Sigma/2 = 0.16(5)$ is considerably smaller than the additive quark model result $\Sigma/2 = 1/2$ of eq. (7). However, the measured spin sum does not only include the contribution of the valence quark spins but also the contribution of quark-antiquark pairs in the Dirac sea as is evident from the definition of Δq in eq. (3). These degrees of freedom are not properly included in the additive quark model in which the axial quark current is approximated as a one-body operator.

3 The PCAC relation and axial exchange currents

When discussing the failure of the additive constituent quark model in correctly describing the axial couplings of the nucleon and the related spin fractions carried by the valence and nonvalence degrees of freedom, it is usually

not mentioned that this model, which uses only one-body axial currents, violates the PCAC relation [16]

$$\mathbf{q} \cdot \mathbf{A}(\mathbf{q}) - [H, A^0(\mathbf{q})] = -i \sqrt{2} f_\pi \frac{m_\pi^2}{q^2 - m_\pi^2} M^\pi(\mathbf{q}). \quad (10)$$

This can be seen as follows. The PCAC relation links the strong interaction Hamiltonian H , the weak axial current $A^\mu = (A^0, \mathbf{A})$ operators, and the pion emission operator described by M^π . Here, m_π is the pion mass and f_π is the pion decay constant for which we use $f_\pi = 93 \text{ MeV}$. If the quark Hamiltonian contains two-body potentials, (*e.g.*, due to gluon exchange), which do not commute with the axial charge density A_0 , the PCAC relation demands that there be corresponding two-body axial currents \mathbf{A} (*e.g.*, axial gluon exchange currents) and two-body emission operators to counterbalance the contribution from the two-body potentials in the Hamiltonian [16]. This is analogous to the requirement of the continuity equation for the electromagnetic quark current [17], which enforces the presence of two-body exchange currents if the two-body potentials do not commute with the quark charge operator.

Equation (10) leads to the Goldberger-Treiman relation [18,19] between the quark axial coupling g_{Aq} , and the pion-quark coupling constant $g_{\pi q}$:

$$g_{Aq} = f_\pi \frac{g_{\pi q}}{m_q}. \quad (11)$$

Here, m_q is the constituent quark mass. In a recent work [16] we have shown that a good description of the nucleon's isovector axial vector form factor $g_A(q^2)$ is obtained if the axial current operator \mathbf{A} satisfies the PCAC condition (10), in particular, if g_{Aq} is determined from eq. (11).

In the present paper, we generalize these results to three flavors, and calculate the axial form factors $g_A^8(0)$ and $g_A^0(0)$ for $SU(3)$ flavor octet baryons in a chiral constituent quark model based on a nonlinear σ -model Lagrangian [20] that is invariant under $U(3)_V \times U(3)_A$ chiral transformations. After an expansion in powers of $1/f_\pi$ the effective Lagrangian reads

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}(i\gamma^\mu \partial_\mu - M')\Psi + \frac{1}{2} \partial^\mu \Phi_j \partial_\mu \Phi_j + \frac{1}{2} \partial^\mu \Phi_0 \partial_\mu \Phi_0 \\ & - \frac{1}{2} m_{\Phi_j}^2 \Phi_j^2 - \frac{1}{2} m_{\Phi_0}^2 \Phi_0^2 - \frac{1}{2} m_{\Phi_{s,0}}^2 \Phi_s \Phi_0 - \frac{3a}{2N_c} \Phi_0^3 \\ & - \frac{1}{2} \text{tr}(F_{\mu\nu} F^{\mu\nu}) - g \bar{\Psi} \gamma^\mu G_\mu \Psi \\ & + \frac{g_{Aq}}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \lambda^j \Psi \partial_\mu \Phi_j + \frac{g_{Aq}^0}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \lambda_0 \Psi \partial_\mu \Phi_0 \\ & - \frac{1}{4f_\pi^2} f_{jkl} \bar{\Psi} \gamma^\mu \lambda^j \Psi \Phi_k \partial_\mu \Phi_l. \end{aligned} \quad (12)$$

Here, Ψ and Φ_0, Φ_j with $j = 1, \dots, 8$ represent the quark and the nonet of pseudoscalar meson fields [21], respectively, G_μ is the gluon field, and $F_{\mu\nu}$ is the gluon field strength tensor. The λ^j are $SU(3)$ flavor matrices,

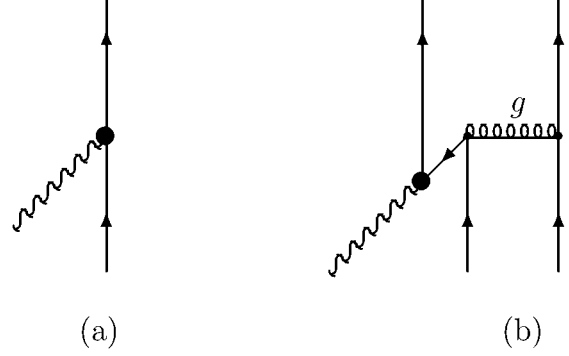


Fig. 1. Feynman diagrams for the axial current operators of constituent quarks: (a) one-body axial current, (b) two-body gluon exchange axial current. The wavy lines correspond to weak gauge bosons coupling to the axial currents. The black dot denotes the axial quark coupling.

$\lambda^0 = \sqrt{2/3} \mathbf{1}$, and the f_{jkl} are the $SU(3)$ structure constants.

Due to the complicated structure of the QCD vacuum characterized by a nonzero quark condensate $\langle 0|\bar{\Psi}\Psi|0\rangle \neq 0$, the chiral symmetry of the Lagrangian is spontaneously broken. This gives rise to nonzero constituent quark mass terms. In addition, there are small QCD quark mass terms which break chiral symmetry explicitly. The matrix $M' = \text{diag}(m_q, m_q, m_s)$ in eq. (12) contains both contributions. The small QCD quark mass terms together with a nonzero quark condensate are also responsible for the meson mass terms $m_{\Phi_0}^2$, $m_{\Phi_j}^2$ ($j = 1, \dots, 8$), and $m_{\Phi_{s,0}}^2$.

The axial or chiral $U(1)_A$ anomaly of QCD is also taken into account in this effective low-energy Lagrangian through an extra mass term $-3a/2N_c$ for the Φ_0 , where N_c is the number of colors and a is an $\mathcal{O}(N_c^0)$ quantity having dimensions of mass square. The axial anomaly breaks $U(3)_F$ flavor symmetry and thus gives rise to different flavor octet and singlet meson masses and axial coupling constants. To take $SU(3)$ flavor symmetry breaking into account, we allow that the hypercharge axial quark coupling g_{Aq}^8 is different from the isovector axial quark coupling g_{Aq} (see below).

From the fundamental interaction vertices of the nonlinear σ model Lagrangian we have derived the potentials and axial current operators of the chiral constituent quark model. The resulting quark potential model contains a gluon exchange potential with strength $\alpha_s = g^2/4\pi$, Goldstone boson exchange interactions with coupling constants $g_{Aq}/2f_\pi$, a confinement interaction, and the corresponding axial currents, as discussed in detail in ref. [22]. For the present purpose it is sufficient to list only the one-body and gluon exchange axial current operators, which contribute to the flavor octet and flavor singlet axial couplings $g_A^8(0)$ and $g_A^0(0)$. These operators are derived from the Feynman diagrams in fig. 1. The one-body isovector axial current operator reads

$$\mathbf{A}_{imp}^j(\mathbf{q}) = g_{Aq} \sum_k e^{i\mathbf{q}\cdot\mathbf{r}_k} \frac{\tau_k^j}{\sqrt{2}} \boldsymbol{\sigma}_k, \quad (13)$$

where $k = 1, 2, 3$ sums over the three constituent quarks in the nucleon, and $j = 1, 2, 3$ denotes the (Cartesian) components of the isospin operator τ_k . The quark spin and position operators are denoted by σ_k and \mathbf{r}_k , and \mathbf{q} is the three-momentum transfer imparted by the weak gauge boson.

To generalize the $SU(2)$ isospin axial current to the $SU(3)$ flavor axial current, we replace the isospin operator τ_k^j by the Gell-Mann flavor operator λ_k^j , where $j = 1, \dots, 8$ is the flavor index. In particular, the $j = 8$ flavor (hypercharge) axial current is obtained by replacing $\tau_k^j/\sqrt{2}$ in eq. (13) by $\sqrt{3}\lambda_k^8$ and g_{Aq} by g_{Aq}^8

$$\mathbf{A}_{imp}^8(\mathbf{q}) = g_{Aq}^8 \sum_k e^{i\mathbf{q}\cdot\mathbf{r}_k} (\sqrt{3}\lambda_k^8) \sigma_k. \quad (14)$$

In the constituent quark model the nucleon consists of u and d valence quarks so that we only need the left upper corner of the $SU(3)$ flavor matrices $\lambda_k^8 = 1/\sqrt{3} \text{diag}(1, 1, -2)$, which is proportional to the unit matrix. Similarly, the flavor singlet one-body axial current is obtained by the replacement $g_{Aq}^8 \rightarrow g_{Aq}^0$ in eq. (14):

$$\mathbf{A}_{imp}^0(\mathbf{q}) \equiv g_{Aq}^0 \sum_k e^{i\mathbf{q}\cdot\mathbf{r}_k} \sigma_k. \quad (15)$$

From the diagram of fig. 1(b) we derive the gluon exchange isovector axial current [16,22]

$$\begin{aligned} & \mathbf{A}_g^j(\mathbf{q}) \\ &= g_{Aq} \frac{\alpha_s}{16m_q^3} \sum_{k<l} \lambda_k^c \cdot \lambda_l^c \left\{ e^{i\mathbf{q}\cdot\mathbf{r}_k} \frac{\tau_k^j}{\sqrt{2}} \left[-i(\sigma_k \cdot \mathbf{r}) \mathbf{q} \frac{1}{r^3} \right. \right. \\ & \quad \left. \left. + \left(3(\sigma_k + \sigma_l) \cdot \hat{\mathbf{r}} \hat{\mathbf{r}} - (\sigma_k + \sigma_l) \right) \frac{1}{r^3} + \frac{8\pi}{3} (\sigma_k + \sigma_l) \delta(\mathbf{r}) \right] \right. \\ & \quad \left. + (k \leftrightarrow l) \right\}. \quad (16) \end{aligned}$$

In eq. (16) λ_k^c denotes the color operator of quark k . The relative coordinate between the two quarks exchanging a gluon is $\mathbf{r} = \mathbf{r}_k - \mathbf{r}_l$ with $r = |\mathbf{r}|$, and $\hat{\mathbf{r}} = \mathbf{r}/r$ is the corresponding unit vector. This two-body axial exchange current is consistent with the PCAC constraint in eq. (10) (see ref. [16]).

As before, the $j = 8$ flavor component of the gluon exchange current is constructed from the isovector current by the replacements $\tau_k^j/\sqrt{2} \rightarrow \sqrt{3}\lambda_k^8$ and $g_{Aq} \rightarrow g_{Aq}^8$ in eq. (16). Finally, the flavor singlet gluon exchange current is obtained from there by the substitution $g_{Aq}^8 \rightarrow g_{Aq}^0$. The hypercharge and singlet axial currents satisfy generalized PCAC relations [22], which are more complicated than eq. (10) due to the chiral anomaly of QCD.

For an interpretation of these operators we remember that the constituent quark model is an effective description of the baryon structure in terms of massive valence quarks, which have the same quantum numbers as the original QCD quarks. Nonvalence quark degrees of freedom, such as quark-antiquark pairs and gluons enter the

model in the form of two-body currents and renormalized quark couplings. Thus, the one-body axial current in fig. 1(a) is the current of noninteracting constituent valence quarks (impulse approximation), whereas the two-body axial current of fig. 1(b) describes quark-antiquark and gluon degrees of freedom resulting from the interaction between these valence quarks.

Concerning the axial quark couplings for the isospin, hypercharge, and flavor singlet currents we recall that the properties of massive, spatially extended constituent quarks are generally different from those of the nearly massless and pointlike QCD quarks. For pointlike QCD quarks and if $U(3)_F$ flavor symmetry is exact, one has [23]

$$g_{Aq} = g_{Aq}^8 = g_{Aq}^0 = 1. \quad (17)$$

In contrast, for constituent quarks, which can be viewed as QCD quarks surrounded by a polarization cloud of quark-antiquark pairs, the axial couplings is expected to be different from unity. In fact, several calculations [23–26] show that due to the cloud of quark-antiquark pairs surrounding a QCD quark, the isovector axial quark coupling g_{Aq} is renormalized from 1 (QCD quark) to about 0.75 (constituent quark). This is consistent with the value $g_{Aq} \approx 0.77$ obtained from the empirical pion-quark coupling constant $g_{\pi q}$ via the Goldberger-Treiman relation in eq. (11). In addition, because flavor symmetry is broken, one expects these axial couplings to be different from each other.

As in the case of the isovector axial quark coupling g_{Aq} we calculate the isosinglet hypercharge g_{Aq}^8 and the flavor singlet g_{Aq}^0 axial quark couplings from the strong ηq and $\eta' q$ couplings and decay constants via generalized Goldberger-Treiman relations. Because the physical η and η' mesons are a mixture of the $SU(3)_F$ octet and singlet representations, the corresponding Goldberger-Treiman relations are more complicated than eq. (11). The mixing scheme [27,28] for the determination of g_{Aq}^8 and g_{Aq}^0 from $g_{\eta q}$ and $g_{\eta' q}$ involves two decay constants f_8 and f_0 different from f_π and two mixing angles Θ_8 and Θ_0

$$\begin{aligned} g_{Aq}^8 &= \sqrt{3} f_8 \left\{ \frac{g_{\eta q}}{m_q} \cos \Theta_8 + \frac{g_{\eta' q}}{m_q} \sin \Theta_8 \right\}, \\ g_{Aq}^0 &= \sqrt{\frac{3}{2}} f_0 \left\{ -\frac{g_{\eta q}}{m_q} \sin \Theta_0 + \frac{g_{\eta' q}}{m_q} \cos \Theta_0 \right\}. \quad (18) \end{aligned}$$

As discussed below, the strong ηq and $\eta' q$ couplings can be calculated from the empirical strong ηN and $\eta' N$ couplings. With the mixing angles and decay constants taken from ref. [28], the axial quark couplings g_{Aq}^8 and g_{Aq}^0 can be obtained from eq. (18). However, it should be noted that there are different forms for the isosinglet Goldberger-Treiman relations in the literature, and there is considerable discussion concerning their validity in view of the chiral anomaly of QCD [29–31].

4 Results and discussion

In the following we present our results for the axial couplings $g_A^8(0)$ and $g_A^0(0)$ of the nucleon. As explained in

Table 1. Results for the axial couplings $g_A^8(0)$ and $g_A^0(0)$. The individual contributions are the one-body current (imp), the gluon exchange current (g), the total current (total), the experimental data (exp) [5]. For the constituent quark axial couplings we use $g_{Aq}^8 = 0.96(44)$ (second row) and $g_{Aq}^0 = 0.46(09)$ (fourth row) as determined from eq. (18). The theoretical error bars in the results for $g_A^8(0)$ and $g_A^0(0)$ are due to the uncertainties in the constituent quark axial couplings.

	imp	g	total	exp
$g_A^8(0)/g_{Aq}^8$	0.99	-0.39	0.60	
$g_A^8(0)$	0.95(± 0.44)	-0.38(∓ 0.17)	0.58(± 0.26)	0.58(10)
$g_A^0(0)/g_{Aq}^0$	0.99	-0.39	0.60	
$g_A^0(0)$	0.46(± 0.09)	-0.18(∓ 0.04)	0.28(± 0.05)	0.33(10)

sect. 2, the nucleon spin fractions can be expressed in terms of these axial couplings.

4.1 The flavor octet hypercharge axial coupling $g_A^8(0)$

The flavor octet hypercharge axial coupling $g_A^8(0)$ of the nucleon is given by the matrix element of the sum of one- and two-body axial current operators $\mathbf{A}^8(\mathbf{q} \rightarrow 0)$ in sect. 3 evaluated between proton wave functions. If the Dirac sea is assumed to be $SU(3)$ flavor symmetric, *i.e.*, $\Delta u_{sea} + \Delta \bar{u}_{sea} = \Delta d_{sea} + \Delta \bar{d}_{sea} = \Delta s_{sea} + \Delta \bar{s}_{sea}$ this coupling measures the fraction of the proton spin that is carried by valence QCD quarks, namely

$$g_A^8(0) = \Delta u_{val} + \Delta d_{val} = 1$$

as in eq. (7). This assumption is however not consistent with the data $g_A^8(0)_{exp} = 0.58(10)$. Thus, in addition to the positive valence quark contribution $g_A^8(0)$ contains a negative contribution coming from a flavor asymmetrical polarized quark-antiquark sea.

In the first row of table 1 we show our results for $g_A^8(0)/g_{Aq}^8$. The one-body axial current gives a contribution $A_1 \approx 1$ because the relevant operator $\mathbf{A}_{imp}^8(0)/g_{Aq}^8 = \sum_k \boldsymbol{\sigma}_k$, is simply twice the total spin operator, and the valence quarks are mainly in relative S -wave states. With the two-body axial current included, we obtain

$$\frac{g_A^8(0)}{g_{Aq}^8} = (A_1 + 2A_2) = 0.6. \quad (19)$$

Here, the A_2 term comes from the two-body gluon exchange current. The latter is negative and reduces the valence quark (one-body current) result by about 40%. Quark-antiquark degrees of freedom effectively described by the gluon exchange current in fig. 1(b) are responsible for this reduction.

This result can also be derived from the general spin-flavor structure of the one- and two-body axial currents in the flavor symmetry limit [32], which for the hypercharge current reads

$$\mathbf{A}_z^8 = a \sum_{i=1}^3 (\sqrt{3} \lambda_i^8) \boldsymbol{\sigma}_{iz} + b \sum_{i \neq j}^3 (\sqrt{3} \lambda_i^8) \boldsymbol{\sigma}_{jz}. \quad (20)$$

Here, the constants a and b are parameters that parametrize the orbital and color space matrix elements.

For $|\mathbf{q}| \rightarrow 0$ and S -waves the axial currents in sect. 3 have the same structure as eq. (20), which when evaluated between $SU(6)$ spin-flavor wave functions gives

$$\frac{g_A^8(0)}{g_{Aq}^8} = (a + 2b). \quad (21)$$

The advantage of a quark model calculation as the one presented here, is that it provides expressions for these parameters in terms of the hypercharge axial coupling g_{Aq}^8 and a gluon exchange current contribution, for example, $a \approx g_{Aq}^8$. Furthermore, we find $b < 0$ because the matrix element of the hypercharge gluon exchange current operator in color space is negative.

To calculate $g_A^8(0)$ absolutely, we need a numerical value for the axial quark coupling g_{Aq}^8 . We derive the latter from the generalized Goldberger-Treiman relations in eq. (18). We use the following relations between the strong η and η' couplings to nucleons and quarks:

$$\begin{aligned} \frac{g_{\eta N}}{M_N} &= \frac{g_{\eta q}}{m_q} (A_1 + 2A_2), \\ \frac{g_{\eta' N}}{M_N} &= \frac{g_{\eta' q}}{m_q} (A_1 + 2A_2) \end{aligned} \quad (22)$$

and first determine $g_{\eta q}$ and $g_{\eta' q}$ from the empirical ηN and $\eta' N$ coupling constants $g_{\eta N}$ and $g_{\eta' N}$, and then obtain g_{Aq}^8 and g_{Aq}^0 from eq. (18). Here, A_1 and A_2 denote one- and two-quark contributions to the strong coupling constants. These are exactly the same as in eq. (19). The reason for this exact correspondence is that the one- and two-quark operators needed to calculate the strong meson-baryon couplings are identical to the ones in the axial coupling calculation [32, 33].

There are different experimental determinations for $g_{\eta N}$ ranging from $g_{\eta N} = 2.1$ [34, 35] to $g_{\eta N} = 2.75$ –4.6 [36]. Flavor $SU(3)$ symmetry predicts $g_{\eta NN} = (3 - 4 \frac{D}{D+F}) g_{\pi NN} \approx 3.6$, where the F and D couplings determined from weak hyperon decays have been used [12]. For $g_{\eta' N}$ we are aware of two experimental determinations from η' photoproduction off nucleons giving $g_{\eta' N} = 1.66$ [37] and $g_{\eta' N} = 1.4 \pm 0.1$ [38]. Taking the range of the experimental values for $g_{\eta N}$ and $g_{\eta' N}$ and the central values of the mixing parameters and decay constants from table 1 in ref. [28]

$$\begin{aligned} f_8 &= (1.26 \pm 0.04) f_\pi, & \Theta_8 &= -21.2^\circ \pm 1.6^\circ, \\ f_0 &= (1.17 \pm 0.03) f_\pi, & \Theta_0 &= -9.2^\circ \pm 1.7^\circ, \end{aligned} \quad (23)$$

as input we obtain from eq. (18) the following values for the axial quark couplings:

$$\begin{aligned} g_{Aq}^8 &= 0.96 \pm 0.44, \\ g_{Aq}^0 &= 0.46 \pm 0.09, \end{aligned} \quad (24)$$

where the errors come from the experimental error bars on $g_{\eta N}$ and $g_{\eta' N}$. In addition, there are uncertainties connected with the validity of the generalized Goldberger-Treiman relations and possible three-body axial currents which we have neglected.

4.2 The flavor singlet axial coupling $g_A^0(0)$

As discussed in sect. 2 the physical interpretation of the flavor singlet axial coupling constant $g_A^0(0)$ is the nucleon spin fraction carried by all quarks, *i.e.*, valence plus sea quarks. To calculate $g_A^0(0)$ in the present model, we evaluate the matrix element of the sum of one- and two-quark flavor singlet axial current operators $\mathbf{A}^0(\mathbf{q} \rightarrow 0)$ in sect. 3 between proton wave functions [39]. We then find

$$\frac{g_A^0(0)}{g_{Aq}^0} = \frac{g_A^8(0)}{g_{Aq}^8}, \quad (25)$$

i.e., the same result as in eq. (19).

However, due to the existence of the axial anomaly in the flavor singlet channel, $U(3)$ symmetry is broken. Consequently, $g_{Aq}^0 \neq g_{Aq}^8$ as reflected by the generalized Goldberger-Treiman relations of eq. (18). The ratio of the constituent quark axial couplings $\zeta = g_{Aq}^0/g_{Aq}^8 = 0.48(24)$ obtained from eq. (24) is positive. Thus, it is at variance with the negative ζ in refs. [40]. Also in view of eq. (17) a negative value for either g_{Aq}^8 or g_{Aq}^0 appears to be rather unlikely. Our result for ζ is in good agreement with the calculation in ref. [23] which gives $\zeta = 0.53$. It would be interesting to calculate the hypercharge and singlet axial quark couplings in the model of ref. [25].

Using eq. (24) we get the results for the one-body and two-body axial current contributions to $g_A^0(0)$ as shown in table 1 (fourth row). Due to the gluon exchange current and the renormalized flavor singlet axial quark coupling, the sum of quark spins in the nucleon is only about 1/3 of the value obtained in the additive quark model. Thus, it is possible to describe the singlet axial nucleon coupling in the framework of the chiral constituent quark potential model provided consistent axial exchange currents are taken into account.

Because of angular-momentum conservation, this reduction of the quark spin is compensated by the orbital angular momentum carried by the same nonvalence quark degrees of freedom. This can be seen from the following qualitative argument. When a u -quark with positive-spin projection emits a spin-1 gluon, its spin flips thereby reducing the fraction Δu of the nucleon spin carried by the u -quarks. The gluon moving in the z -direction eventually annihilates into a quark-antiquark pair as in fig. 1(b). These sea quarks have momentum components transverse

to the z -direction and therefore contribute to the orbital angular momentum of the system. In this way the gluon emission and annihilation process described by the two-body exchange current of fig. 1(b) leads to a redistribution of the angular momentum from quark spin to the orbital angular momentum carried by quark-antiquark pairs [41].

This interpretation is supported by the results obtained from pion electroproduction in the Δ -resonance region, in particular, from the $N \rightarrow \Delta$ transition quadrupole moment. The latter is a measure of the deviation of the nucleon charge density from spherical symmetry and therefore of the amount of orbital angular momentum in the nucleon. It was found that the $N \rightarrow \Delta$ quadrupole transition involves mainly quark-antiquark degrees of freedom [42,43], *i.e.*, the same exchange current diagram of fig. 1(b) that leads here to the reduction of the quark spin contribution to the nucleon spin. Thus, the electromagnetic $N \rightarrow \Delta$ quadrupole transition provides an independent indication that quark-antiquark degrees of freedom carry most of the orbital angular momentum in the nucleon.

5 Summary

We have investigated the axial form factors related to the spin structure of the nucleon in the framework of the constituent quark model. In addition to the usual one-body axial current, we have included two-body axial currents as required by the PCAC condition. These two-body exchange currents provide an effective description of the nonvalence quark degrees of freedom in the nucleon. We have shown that they reduce the quark contribution to nucleon spin.

In particular, for the flavor octet hypercharge axial coupling $g_A^8(0)$, we obtain $g_A^8(0) = 0.58 \pm 0.26$, where the uncertainty comes mainly from the hypercharge axial quark coupling $g_{Aq}^8 = 0.96 \pm 0.44$. The latter is determined from the experimental ηN and $\eta' N$ couplings via generalized Goldberger-Treiman relations and reflects the experimental error bars of these coupling constants. We emphasize that the reduction of the additive quark model $g_A^8(0) = 1$ is predominantly due to the negative contribution of the quark-antiquark degrees of freedom effectively described by the axial gluon exchange current.

Also in the case of the singlet axial coupling $g_A^0(0)$, measuring the quark and antiquark spin contribution to the nucleon spin, we find $g_A^0(0) = 0.28 \pm 0.05$, *i.e.*, a drastic reduction compared to the standard additive quark model result $g_A^0(0) = 1$. This is partly due to the negative contribution of the axial gluon exchange current, and partly due to the renormalization of the singlet axial constituent quark coupling $g_{Aq}^0 = 0.46 \pm 0.09$ as determined from the generalized Goldberger-Treiman relations. Both effects lead to a result in good agreement with the experimental value $\Sigma = 0.33(10)$.

Thus, the failure of the usual constituent quark model to describe the spin structure of the nucleon ("spin crisis") is mainly a consequence of the incorrect assumption

that the axial nucleon current is the sum of three one-body quark currents, underlying most applications of this model. We have pointed out that this assumption is in conflict with the PCAC condition, which demands that i) the axial current operator contains two-body axial exchange currents, and that ii) the constituent quark axial couplings are renormalized from their QCD quark values. In summary, including axial exchange currents and assuming the validity of generalized Goldberger-Treiman relations allows a constituent quark model description of the hypercharge and singlet axial couplings of the nucleon that is consistent with the data.

This does not mean that all problems concerning nucleon spin structure are solved. There are important conceptual problems, in particular, the validity of the generalized Goldberger-Treiman relations in the presence of the axial anomaly [29–31], and the relativistic definition of spin itself [44] which we have not addressed. Finally, it would be interesting to calculate the orbital angular momentum associated with the axial gluon exchange current investigated here.

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9. In the following, we use the name “QCD quark” instead of “current quark” to denote the nearly massless quark fields appearing in the QCD Lagrangian.
10. A description in which the axial current contains only one-body operators is often referred to as “impulse approximation”.
11. Due to the axial gluon anomaly of QCD, gluon spin contributions $\Delta G(Q^2)$ are admixed to the quark spin contributions in leading-order perturbation theory. As a result, the deep inelastic scattering experiments actually measure $\Delta q(Q^2) = \hat{\Delta}q - \alpha_S(Q^2)\Delta G(Q^2)$, where α_S is the running QCD coupling constant. Thus, the Q^2 -dependence cancels in the quark spin differences contained in $g_A(0)$ and $g_A^8(0)$ but remains in the quark spin sum $g_A^0(0)Q^2$. This Q^2 -dependence is very soft in the perturbative regime, but its evolution down to the confinement scale is not known.
12. From neutron β -decay one can extract $g_A(0) = 1.2670 \pm 0.0035$ [13]. Similarly, from the β -decay of Ξ^- -hyperon, and the assumption of $SU(3)$ flavor symmetry [14,15] one obtains $g_A^8(0) = 0.588 \pm 0.033$ (see ref. [5] and references therein). Instead of the axial couplings $g_A(0)$ and $g_A^8(0)$, which govern the β -decay of octet baryons in the $SU(3)$ limit, the symmetric and antisymmetric flavor octet coupling constants D and F are often used. The relation between both notations is $g_A(0) = F + D$, and $g_A^8(0) = 3F - D$.
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